Re-focussing Research Agendas

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Numerous recent historical and current developments have occurred that impact on the nature of mathematics and mathematics education, and education in general. In this paper some of these are discussed as they suggest possible new foci for research.

Introduction

Research is an interesting and ever-changing activity. Some research projects have small foci that contribute to larger bodies of knowledge, some are more important than others, and some projects that seem to have potential are ignored. As a mathematics-education community we need, from time to time, to step back from our current projects and reconsider the research agenda in terms of what are the foci that are most likely to contribute to the further development of our discipline.

In this paper I present some of my own ideas on research foci based on my scholarship and interests, in doing so I am not intending to change minds, but rather to stimulate debate about where we might be going. I believe that too often we focus on ways of improving what is currently happening and on developments that are related to the current situation, rather than considering alternatives. This is understandable as the way research funding is allocated means that neither *blue sky* research that considers possible future trends nor *critical* research that questions underpinning assumptions or explores alternatives on which our research is or could be based are prioritised.

Recent History

Consider three examples from our recent history: assessment, learning and teaching theories, and the basic building blocks of mathematics. In two of these I have seen little significant change occurring, and with the other some change is starting to emerge, though I wonder if the change is as radical as it might be.

Assessment

The emphasis on assessment in education occurred with the acceptance of behaviourism as the dominant learning theory. Subjects were broken down into small measurable objectives that were generally thought of sequentially. Assessment of these was reinforced when educational policy makers, mistakenly I believe, implemented approaches to assessment based on production line approaches to quality management.

With behaviourist views of learning and teaching outmoded by constructivist perspectives, little research seems to have occurred on what might be called constructivist modes of assessment, nor have alternative quality measures been investigated. The result of this is that assessment, testing, and much of our research remains underpinned by behaviourism; and this in turn influences teaching and curriculum. Personally I doubt if the behaviourist influence is desirable.

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Another aspect of assessment that is reinforced by government-speak to parents in terms of their right to know that the very best is done for their children. However, the very best that happens in schools is much more than the subjects we teach yet this is not usually assessed. No one seems to be researching whether assessment has any ill-effects on low-achieving children. I wonder if assessment results tell 'less successful' students that they do not know and cannot learn and in fact teach them that they are failures. We also need to consider whether this assessment encourages competition rather than cooperation, and is this what we want.

Further concerns with assessment relate to the fact that some of the new curriculum documents list competencies and thinking and aspects of personal and social development are amongst these. However, there is no indication within curriculum that these competencies should be assessed, only subject matter. Perhaps, from the perspective of Lipman (2003), personal and social development might be given more consideration if we were to have a *caring* curriculum, but unfortunately in mathematics education we seem to have a *critical* rather than a *caring* or a *creative* curriculum. In addition, we know that thinking includes meta-cognitive thinking or self-monitoring; and this suggests that we should be encouraging self-assessment. I believe that the competencies offer many opportunities for research, assessment being just one such area.

Learning and Teaching Theories

I have mentioned behaviourism and constructivism but there are more theories about learning and teaching. In 1999 I presented a paper (Begg, 1999) in which I began to consider theories alphabetically—associationism, behaviourism, constructivism and so on. As I extended this I realized my error, I was wanting a *best* theory. I gradually realized that no theory is true, each has something to offer, if it did not then it would never have emerged, and all theories are socially and culturally contextual. Now, in terms of theories, I ask 'what is the *x*-factor in learning and teaching?' (see table 1).

With this in mind I wonder why so much of our current mathematics education research is based upon a constructivist philosophy, and why so little is done to explore what other theories might contribute to our knowledge of mathematics education.

Basic Building Blocks

Numeracy has become a political catch phrase during this decade in mathematics education in New Zealand and Australia with more research and development funding being allocated to numeracy than to the rest of mathematics. This situation is based on an assumption, supported, not unreasonably, by those with vested interests, that numeracy is the basic building block of mathematics. I, and numerous colleagues, find this assumption problematic. When the assumption is critiqued the numeracy supporters make claims that numeracy means all of mathematics, but their practices do not support this defensive statement. No doubt number is important, but I do not think it deserves centre stage.

Recently Mulligan and colleagues have recently been working on a project that emphasises structure and pattern (Mulligan, 2009) and this goes some way towards questioning the centrality of number. Hearing about this initiative reminded me of the 1960s and the *new math* movement when mathematicians also focused on mathematical structure, pattern and logic. From my perspective the new mathematics focus in primary schools was on sets and operations, while at high school it was on relations and mathematical systems. While numbers were amongst the elements discussed as students learnt about sets, operations, relations and systems, there were also other elements that enriched the students' views of mathematics.

Table 1. *What is the X-factor in Learning and Teaching?*

Activity-based learning, activity theory, alternative education, appreciative inquiry, apprenticeship, artificial intelligence, associationism, authentic learning, awareness
Behaviourism, bodily knowing
Caring, cognitivism, communities of practice, complexity, connectionism, constructionism, constructivism, cooperative/collaborative learning
Darwinism, Deweyism, direct instruction, discovery learning, drill and practice
Elaboration theory, e-learning, emancipatory learning, emergence, enactivism, experiential learning
Facilitative teaching, Friere's critical education
Gestalt, group learning, growth theory
Hands-on learning, hermeneutics, holistic learning, humanist education
Imitative learning, individualised learning, integrated learning, intuitional learning
Job-based learning
Kinaesthetic education, koan-based learning
Lecture-based learning, living is learning
Meditative practices, Montessori education, motivational theories, multiple intelligences
Narrative pedagogy, neo-behaviourism, neo-Deweyism
Observation-based learning
Phenomenology, Piagetian development, post-structuralism, problem-based learning, progressive education
Question-based (Socratic) learning
Radical constructivism, reciprocal learning-teaching, reflective practice, Roger's 'freedom to learn', Rousseau's natural education
Schema theory, self-directed learning, situated cognition, social constructivism, structuralism
Tabula rasa, thinking-based learning, training, transformative learning, transpersonal consciousness
Unconscious knowing/learning
Visual learning, vocational education, Vygotsky's socio-cultural constructivism
Waldorf/Steiner education, women's ways of knowing, writing-based learning
X
Yin-yang learning
Zone of proximal development

The numeracy projects, like the new maths initiatives, seem concerned with sequential learning of behavioural objectives and these objectives are usually based on content (facts and simple procedures). This emphasis continues in spite of the 1990s initiatives following the NCTM standards (NCTM, 1989) when the traditional content aspects of many curriculum documents were supplemented by the mathematical processes (problem solving, communicating, logical reasoning, and making connections); and the current round of curriculum development in which the competency thinking is introduced. Thus, while curriculum documents suggest that we focus on and integrate content, processes, and thinking, the numeracy projects continues to take a narrow approach.

Current Developments

A number of challenges arise for mathematics education at school and tertiary level as we think more deeply about the nature of mathematics. These include the place of applied mathematics and knowledge modes, the role of technology, and mathematical thinking.

Applied Mathematics and Knowledge Modes

Traditionally applied mathematics was taught at the tertiary level as a complement to pure mathematics, and this was reflected in some high schools in the 1960s when mechanics was called applied mathematics and then later when applied mathematics developed to include statistics and computing. In New Zealand schools some emphasis was put on mathematical applications and at the senior level pure and applied mathematics were changed to mathematics with calculus and mathematics with statistics because all mathematics was seen to have applications. (At the same time mechanics was considered to be part of physics, and computing became a subject in its own right.) However, although many advances in mathematics are made when needs arise within practical situations, in schools applications seem to be taught as an afterthought and not as a justification for the inclusion of topics.

At a recent seminar a colleague (Kell, 2009) discussing *literacies* talked about *modes of knowledge.* She defined mode one as traditional, privileged, canonical (or official), academic mode, and mode two as what is required for problem solving and involves interpretation, cross-disciplinary knowledge, and sees meaning as being context dependent.

Thinking about applied mathematics and knowledge modes, it seems that we need to confront the question, should mathematics (at any or every level, pre-school, primary, secondary or tertiary) be mode one knowledge or mode two knowledge? The way that mathematics has emerged is a result of a need and the way that mathematics can be taught through applications rather than having them as an add-on suggests that perhaps we need to research 'mode two' mathematics. Related to mode two knowledge is the notion of *incidental* learning and this is another area that might be added to our research agenda.

Role of Technology

Technology is changing our world, including the way that mathematics is done. Simple number work can be done with a four-function calculator. Nearly every algebraic and differential equation that a student meets at school and in their undergraduate years can be easily solved with a computer. Many geometry tasks can be explored and results demonstrated with dynamic geometry software. Statistics software offers new approaches to handling data. While technology offers all of this we continue to teach most of our mathematics as if technology did not exist.

For me the mathematical processes suggested making connections and using tools and these imply embracing calculators and computers. Embracing technology involves a revolution in how we see the nature of mathematics and mathematics education; it suggests a shift in emphasis to ideas (thinking) rather than facts and procedures (content); it will require a significant change in the way mathematics is taught, learnt, and assessed, and we need to experiment with and research this change of focus towards mathematical ideas.

Mathematical Thinking

In considering content, processes, and thinking (or knowing, doing and thinking) as three aspects of mathematics there is no intention to suggest that these are separate; indeed, one cannot think or do without having something to think about or use. I envisage knowing, doing and thinking as three sets that are nearly completely overlapping, and as three aspects of our subject that offer three ways of looking at mathematics—though, until recently we seem to have concentrated mainly on content and process even though some colleagues (for example, Mason, with Burton & Stacey, 1982) have emphasised thinking.

While a Venn diagram with overlapping sets is how we might visualise these three aspects of mathematics, I have found it useful to draw an equilateral triangle with the vertices knowing, doing and thinking, and to use this triangle as a 'map' on which we can position classroom tasks and activities as we reflect on the focus of the three aspects in both our intended lesson planning and in the way that the lesson eventuated. Having such a focus on the emphasis and opportunities afforded by tasks and activities for classes seems to me an important aspect of lesson planning and again a possible area for research.

Thinking in general is usually considered from three perspectives, critical, creative, and meta-cognitive. Washburn (2009) uses terms such as comparing, generalizing, reasoning, explaining, and justifying in his discussion of critical thinking and all mathematics teachers are compared with these ways of thinking. Meta-cognitive thinking includes self-monitoring and, as already mentioned, this includes (but is not limited to) self-assessment. Creative thinking is another aspect of mathematics that is often ignored. One aspect of creative thinking is visualising and with just this one focus there are many opportunities for research. For example, have we discouraged visualisation by always giving diagrams in geometry text book problems, and have we limited visualisation by using squared paper with, typically, all the vertices of a 'general' triangle been marked on the grid intersection points.

Now that thinking has been mentioned in curriculum documents as a competency (or aim) of education perhaps our research agenda might move to explore what it means in mathematics education. I believe that such research will enrich mathematics education at all levels.

Mathematics Education

In terms of teaching and learning, I love the theorem, "Teaching is neither a necessary nor sufficient condition for learning." I am sure that all of you can find counter examples from your own experience to demonstrate the truth of both 'not necessary' and 'not sufficient'. Of course this theorem does not say that teaching cannot assist the learning process, but perhaps we need to explore and research both teaching and learning from broader perspectives than we have used in the past. As part of such an exploration it seems useful to consider pre-school learning, informal learning, and what we mean by learning.

Pre-school Learning

We have much to learn from pre-school children. Without being formally taught, pre-school children immersed in multi-lingual contexts develop significant vocabularies and informal (but high-level) rules for grammar in each of the languages they are exposed to. I would suggest that at the same time they also learn much more mathematics and mathematical language than we ever give them credit for. They learn about: mathematical relations in a very general way (brother, sister, mother father), order relations (older, bigger, stronger, etc), size (bigger, smaller, and same), operations (adding and taking away, firstly in terms of quantities rather than numbers), shapes and properties (3-d before 2-d, curves, straight), quantity (few, lots, matching), comparing, and counting. This mathematics that they learn, like the vocabulary and grammar rules they learn, is for practical purposes. Perhaps if learning for a purpose is so successful then school mathematics should focus more on real world situations with the mathematics being learnt incidentally as mode two learning.

Informal Learning

I imagine that most of you, like me, have in the last two or three years learnt to solve a sudoka task. Such tasks are often said to be non-mathematical, and certainly one can design them using letters, pictures, colours, or whatever in place of the numbers. However, they are problems that require the development of problem-solving strategies. I am interested in how you learnt to complete such tasks? I wonder about the strategies you used after the initial trial and error exploration? Could you explicitly list these strategies? Did you think of the strategies suggested by Polya (1957)? I doubt if you went to a sudoka class, or were even taught, though perhaps someone shared a hint or two. I wonder too, have you progressed past the easy and medium, do you enjoy the challenge of the hard, and have you decided that the diabolical are just too tough?

Learning to solve sudoka tasks is an experience that most of us have had of informal learning. The task provided the motivation, though in fact it was little more than a sophisticated way of wasting time. Much of the learning was 'incidental' to the task and only became specific if one reflected on what you had done or considered the task very consciously from a meta-cognitive perspective. Informal learning, like pre-school learning, involves mode-two knowledge and much of what happens is incidental to the task. My question is, does this type of learning feature in our research agendas?

Education

To be successful as educators we need to know what we think successful learning entails, what are our responsibilities beyond subject teaching, and how, within a system dominated by imposed curriculum documents, might we achieve success.

What is Learning?

I believe that the question 'what is learning?' is not asked enough. Is recall of facts (memory work) learning? And is it still learning if little understanding is associated with the facts? I can remember being able to produce proofs in classes by memory with very little understanding of the logic behind them—was this learning? And most importantly, is learning to know, do, and think all we need for mathematics education. While each of us may have our own opinions about what constitutes learning, I wonder if our opinions are shared or communicated to our students, to their parents, and to educational administrators. Perhaps this is an area of policy research that could be considered.

Aims of Education

For me, mathematics (like other subjects) is situated within a broad educational context, and each subject has aims which are secondary to the more general aims of education. In my early teaching career a report was published that outlined a set of aims for education that immediately appealed to me. Over the years I have often returned to these aims though I acknowledge that I now interpret them at a much deeper level than I did when I first read them. The aims were (Munro, 1969, p. 1):

> the greatest emphasis should be put on fostering
> the urge to enquire
> the desire for self respect
> a concern for others

Now, nearly forty years later, a new school curriculum has been published in New Zealand (Ministry of Education, 2007), and, as in numerous other countries, it includes 'competencies' instead of aims. My immediate reaction was to notice the similarity between the 1969 aims and these competencies. They both map onto the three domains of growth that concern teachers. These three domains can be represented in different ways. If one sees them as mind, body, and society then the best representation might be a Venn diagram with three nested sets (and this can be extended with an additional set for the eco-system). However, the nested sets suggest if one works on the inner set then one is also working on the others. I prefer another triangular map with the intellectual, personal and social domains at the vertices.

It is interesting to consider the tasks we use with our students in mathematics classes, other subject classes, and extra-curriculum activities, and consider where these tasks are and where they might be positioned on such a map. Initially I mapped many mathematical tasks close to the intellectual vertex, but then realized that the classroom pedagogy usually involved personal and social growth. However, I was reminded of the saying that "primary school teachers teach children, high school teachers teach mathematics" and I wondered whether my pedagogical approaches facilitated personal and social growth as much as I hoped.

Using this 'map' can help us note whether the tasks we use with our students give too much emphasis to the intellectual domain at the expense of the other two; and to think of our work in terms of facilitating personal and social growth. I see integrating the personal and social aspects of growth with intellectual growth as acknowledging the nested nature of mind/body/society. If we only emphasise the intellectual aspect then our subject is likely to be seen by many students as disconnected from them and society. However, how such integration might be achieved is a challenge and perhaps some exploratory research might assist teachers in changing their emphasis.

Using Curriculum

Some teachers see curriculum statements (be they school-based, or regional/national) as restrictive; others see them as opportunities. Some concentrate on note the mathematical content lists while others try to implement the 'spirit' of the curriculum. Some see curriculum documents as planning documents on which course planning is based. A few teachers take the alternative view, they build up the course they are teaching using rich learning tasks and activities that they have found stimulate good learning, then use curriculum documents as a checklist to identify remaining gaps. How curriculum documents are used in terms of specific subject planning, and in terms of the aims/competencies of education is not well documented and this suggests another area for research.

Conclusions

Overall my feeling is that sometimes we are so busy researching specific concerns that we do not consider the big picture, and often our immediate concerns actually cause us to move our attention from the bigger issues. I see mathematics education as being in a period of exciting change and I hope that our research endeavours embrace and address these changes. If we do not face these changes then I fear that mathematics might become the Latin of the 21st century.

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